Flexible Multidimensional Scaling for Human Smart Development Analysis in EU Countries

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Abstract:

Purpose: The aim of the paper is to propose a flexible procedure to Multidimensional Scaling, allowing to calculate the input distance matrix based on slightly different set of variables for each pair of objects.

Design/Methodology/Approach: The procedure starts from the classical standardization of each variable. Before the calculation of flexible distance between two objects, we eliminate the variable with the biggest absolute value in the first object, and the same we do for the second object. So, we have two variables less in the list for these two objects. If by chance the same variable is pointed for elimination by both objects, the next variable with the biggest (out of both objects) absolute standardized value should be eliminated. With this procedure, each element of distance matrix is based on the same number of variables, but some of actual variables can be different.

Findings: As an example – Flexible Multidimensional Scaling is performed on the list of 17 variables describing so called smart society, for 28 European Union countries. It shows how the proposed procedure works in practice.

Practical Implications: The proposed flexible procedure can be used for the analysis of any problem treated by Multidimensional Scaling.

Originality/Value: Flexible Multidimensional Scaling (FMDS) is a new idea and method. It eliminates some elements of subjective choice of initial variables and seems to be more robust against outliers than classical Multidimensional Scaling (MDS).

Keywords: Multidimensional scaling, flexibility, smart society, European Union countries.

JEL codes: C39, I21, O52, O57.

Paper type: Research paper.

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1. Introduction

The flexibility idea has been introduced to the multivariate data analysis by Drewnowski (1966). He suggested flexibility criterion as one of the ways to evaluate methods for linear ordering of multidimensional objects. The classification is flexible if the slight change in the list of variables (by adding or omitting variables) does not strongly affect the final ordering of objects. In some applications – especially in economics – extremely skewed variables can influence results strongly, making research vulnerable to variable choice decision.

Multidimensional Scaling (MDS) is a procedure which allows to present the configuration of objects or variables from multidimensional data in lower-order space – usually two-dimensional, on the plane. The configuration on the plane should produce a distance matrix most like the distance matrix calculated in the original multidimensional space. This similarity is usually assessed by STRESS measure proposed by Kruskal (1964). There are many books describing in details - concepts, methods and algorithms used in MDS, e.g. (Kruskal and Wish, 1978; Schiffman *et al.,* 1981; Young, 1987; Borg and Groenen, 2010; Ding, 2018). Interesting review of basic MDS ideas and literature until 1980 can be found in Davinson (1983).

Complex phenomena are naturally multidimensional, and can be analysed from different points of view, but the general, overall approach is usually more attractive. In social science and economics such categories as development, welfare, poverty, quality of life, human and intellectual capital, globalization, economic crises are examples of such phenomena. They cannot be expressed by just one variable, and are obvious subjects for multivariate analysis such as linear ordering, composite indicators, cluster analysis, classification and dimensionality reduction.

Such multivariate approach has been shown by A. Murawska and others (Murawska *et al.,* 2020) in multidimensional analysis of the relationship between sustainable living conditions and long life in good health, in EU countries. A methodology for the study of multidimensional and longitudinal aspects of poverty and deprivation was proposed by G. Betti and V. Verma (Berri and Verma, 2004). Gumpert (2019) analysed the extension of the core-peripheral Krugman model toward the inclusion of many economic aspects. This paper is the continuation of research on educated and smart society development. Education and human capital were discussed by many notable scientists, such as T.W. Schultz (1961; 1981), G. Becker (1962; 1993), A. Sen (1993; 1994; 1999; 2002), and Borowiecki *et al.* (2021).

The results and quality of multivariate analysis heavily depend on the choice of variables. This choice is always - to some extend - subjective, and usually limited by the availability of data. So why, studies on flexibility which is understood as the robustness against small changes in the list of diagnostic variables, seems to be important. We have already proposed the flexible approach to cluster analysis (Sokołowski and Markowska, 2021). Many analyses have been done to the European

Union countries with dynamic cluster analysis (Markowska *et al.,* 2021). Multidimensional Scaling provides interesting, and rather simple, graphical illustration for multidimensional problems, so introducing flexibility aspect looks promising here. In this paper we used the proposed methodology to analyse educated and smart society development in the European Union.

2. Research Methods

Multidimensional Scaling is finally performed on distance matrix, and the knowledge of original data is even not mandatory. But in the proposed procedure we start from the data on all variables. While calculating distances between pairs of objects, two variables (one for each object) with most outlying values are eliminated. The proposed approach can be defined in the following steps:

- Define the set of objects to be analyzed,
- Set the initial list of m variables,
- Standardize variables,
- Calculate distance matrix $\mathbf{D}$ as "per variable" taking d_{lk}/m,
- Perform MDS on m variables,
- For each object j find variable i^* with the highest module of standardized value and variable i^{**} with the second highest module,
- Calculate distance matrix $\mathbf{D^*}$ with distance between objects l and k is calculated using m-2 variables, omitting variables i^*_j and i^*_k,
- If both objects l and k point out the same variable i^*, select one variable with the higher module, from i^{**}_l and i^{**}_k (it will be the second variable eliminated),
- Divide each element of $\mathbf{D}^*$ by m-2, making it as "per variable",
- Perform MDS on $\mathbf{D}^*$ distance matrix.

The degree of flexibility (of the MDS based on original list of variables) can be evaluated by measure M. It compares distance matrix $\mathbf{D}_{MDS}$ calculated for the two-dimensional MDS result based on original full list of variables with $\mathbf{D}^*_{MDS}$ matrix based two-dimensional MDS result calculated using flexibility procedure. The degree of flexibility measure (M) takes value between 0 and 1 – the bigger the more flexible are results of the original MDS (less vulnerable to omitting some variables):

$$M = 1 - \frac{\sum_{l=1}^{n}\sum_{k=1}^{n}(d_{lk}-d_{lk}^*)^2}{2n^2}, \tag{1}$$

where:
n – number of objects,
l – row number in a distance matrix,
k – column number in a distance matrix,

d_{lk} – elements of distance matrix of two-dimensional configuration of points resulted from MDS based on full list of variables,

$d_{lk}*$ – elements of distance matrix of two-dimensional configuration of points resulted from MDS based on flexibility procedure.

All calculations presented in this paper were done on *STATISTICA ver. 13* software.

3. Research Data

In this paper, flexible multidimensional scaling (FMDS) is illustrated by the analysis of 28 European countries characterized by 17 variables describing human smart development. The data covers year 2017. Here is the list of variables (https://strateg.stat.gov.pl/dashboard/#/56 polityka-spojnosci/1):

X_1 – Expenditures on R&D as % of GDP,

X_2 – % of population using internet at least once a week,

X_3 – Number of inventions reported to EPO (European Patent Office) per 1 million population,

X_4 – PISA (Programme for International Student Assessment) test – % of students on high levels in reading and interpretation,

X_5 – HDI (Human Development Index),

X_6 – PISA test - % of students on high levels in mathematics,

X_7 – PISA test - % of students on high levels in science,

X_8 – Government and higher education sector expenditures on R&D as % of GDP,

X_9 – Corporate expenditures on R&D as % of GDP,

X_{10} – % of small and medium enterprises adopting product or process innovations,

X_{11} – % of population with high internet skills,

X_{12} – % of employees working in R&D,

X_{13} – % of young people not working nor learning,

X_{14} – % of teenagers who quit education,

X_{15} – % of 25-64 population still active in education,

X_{16} – % of 30-34 population with higher education completed,

X_{17} – % of population with at least basic computer skills.

Most of the variables used in the analysis are stimulants (the bigger, the better), and only two (X_{13} and X_{14}) are destimulants (the smaller, the better). Basic descriptive statistics of variables describing human smart development are presented in Table 1 (Part 1 and 2).

Countries with the biggest number of "the best" values (first or second maximum of stimulants, first or second minimum of destimulants are: Sweden (eight variables: X_1, X_2, X_3, X_8, X_9, X_{11}, X_{13}, X_{15}), Finland (four variables: X_4, X_7, X_{12}, X_{15}), Denmark (three variables: X_2, X_3 and X_8) and Holland (three variables: X_6, X_{13}, X_{17}). Two variables

with the best values were observed in Estonia (X_6–X_7), Ireland (X_4 and X_5), Lithuania (X_{11}, X_{16}), Luxembourg (X_2, X_{17}) and Austria (X_1 and X_9). Definitely the biggest number of "the worst" values (first or second minimum in stimulants, first or second maximum in destimulats) characterize Romania – 14 variables (X_1, X_2, X_4–X_8, X_{10}–X_{12}, X_{14}–X_{17}). There were seven such variables in Bulgaria (X_2, X_3, X_5, X_8, X_{12}, X_{15} and X_{17}), and three in Latvia (X_1, X_9, X_{10}). Two less favourable variable characterize Malta (X_9, X_{11}), Italy (X_{13}, X_{16}), Croatia (X_3, X_{17}) Greece (X_7, X_{13}) and Cyprus (X_6, X_7).

Table 1. Descriptive statistics of variables – part 1

Variable	Max 1	Country	Max 2	Country	Min 1	Country	Min 2	Country
X_1	3.33	SE	3.16	AT	0.50	RO	0.51	LV
X_2	96.00	LU	95.00	SE, DK	61.00		62.00	BG
X_3	283.46	SE	246.61	DK	4.13	BG	4.80	HR
X_4	13.90	FI	13.80	IE	1.90	RO	2.50	SK
X_5	0.94	IE	0.94	DE	0.81		0.81	BG
X_6	15.00	EE	14.80	NL	3.45	CY	3.60	RO
X_7	13.70		13.10	FI	0.60	RO	1.80	EL, CY
X_8	0.97	SE	0.95	DK	0.21		0.21	BG
X_9	2.35		2.22	AT	0.00	MT	0.14	LV
X_{10}	57.95	BE	47.87	PT	2.57		11.78	LV
X_{11}	42.00	SE	36.00	LT	6.00	RO	6.40	MT
X_{12}	3.74	EL	3.24	FI	0.53		0.86	BG
X_{13}	25.50	IT	24.20	EL	6.90	SE	7.30	NL
X_{14}	18.30	ES	18.10	RO	3.10	HR	4.30	SI
X_{15}	30.40	SE	27.40	FI	1.10	RO	2.30	BG
X_{16}	58.00	LT	55.90	CY	26.30		26.90	IT
X_{17}	85.00	LU	79.00	NL	29.00	BG, RO	41.00	RO

Source: Own calculations based on data from Statistics Poland, https://strateg.stat.gov.pl/dashboard/#/56 polityka-spojnosci/1.

Table 1. Descriptive statistics of variables – part 2

Variable	Median	Average	Standard deviation	Coefficient of variation	Max/min	Max-min
X_1	1.31	1.57	0.88	56.02	6.66	2.83
X_2	79.50	80.14	10.11	12.61	1.57	35.00
X_3	34.67	82.88	91.76	110.72	68.63	279.33
X_4	6.15	7.21	3.53	49.03	7.32	12.00
X_5	0.88	0.88	0.04	4.25	1.16	0.13
X_6	10.25	9.83	3.69	37.56	4.35	11.55
X_7	6.65	6.72	3.53	52.52	22.83	13.10
X_8	0.56	0.57	0.24	41.82	4.62	0.76
X_9	0.71	0.97	0.67	68.86	-	2.35
X_{10}	32.32	29.57	12.91	43.65	22.55	55.38
X_{11}	15.50	16.75	8.75	52.24	7.00	36.00
X_{12}	2.07	2.03	0.83	40.96	7.06	3.21
X_{13}	13.15	13.95	4.76	34.10	3.70	18.60

X_{14}	8.70	9.41	4.03	42.84	5.90	15.20
X_{15}	9.40	11.32	7.78	68.78	27.64	29.30
X_{16}	44.05	42.09	9.02	21.44	2.21	31.70
X_{17}	56.00	56.91	13.92	24.46	2.93	56.00

Source: *own calculations based on data from Statistics Poland.*
https://strateg.stat.gov.pl/dashboard/#/56 polityka-spojnosci/1.

Values of percentage coefficient of variance greater than 50, allow to identify variables mostly differencing EU countries, as: number of inventions reported to EPO per 1 million population, corporate expenditures on R&D as % of GDP, % of 25-64 population still active in education, expenditures on R&D as % of GDP, PISA test – % of students on high levels in science and % of population with high internet skills. Clearly, the smallest variability has been found in HDI. It is also backed by the relations of maximum to minimum for this variable. In Table 2, variables which were identified to be eliminated while calculating matrix D*, are listed for each EU country. Plus indicates that the standardized value under the module was positive, and minus that it was negative.

Table 2. *Extreme value variables for EU countries*

Country	Standardized variable with the highest absolute value	Standardized variable with the second highest absolute value
Austria	X_9 – Corporate R&D expenditures (+)	X_1 – Expenditures on R&D (+)
Belgium	X_{10} – SME with innovations (+)	X_9 – Corporate R&D expenditures (+)
Bulgaria	X_{17} – Computer skills (-)	X_5 – HDI (-)
Cyprus	X_6 – PISA mathematics (-)	X_{16} – 30-34 with higher education (+)
Czechia	X_6 – PISA mathematics (+)	X_{11} – High internet skills (-)
Germany	X_9 – Corporate R&D expenditures (+)	X_1 – Expenditures on R&D (+)
Denmark	X_{15} – Adults active in education (+)	X_3 – Inventions in EPO (+)
Estonia	X_7 – PISA science (+)	X_6 – PISA mathematics (+)
Spain	X_{14} – Teenagers quitting education (+)	X_4 – PISA reading and interpret. (-)
Finland	X_{15} – Adults active in education (+)	X_4 – PISA reading and interpret. (+)
France	X_4 – PISA reading and interpret. (+)	X_{11} – High internet skills (-)
Great Britain	X_2 – Internet activity (+)	X_7 – PISA science (+)
Greece	X_{13} – Young not working/ learning (+)	X_{12} – Employment in R&D (+)
Croatia	X_{14} – Teenagers quitting education (-)	X_2 – Internet activity (-)
Hungary	X_5 – HDI (-)	X_{16} – 30-34 with higher education (+)
Ireland	X_4 – PISA reading and interpret. (+)	X_5 – HDI (+)
Italy	X_{13} – Young not working/ learning (+)	X_{16} – 30-34 with higher education (-)
Lithuania	X_{11} – High internet skills (+)	X_{16} – 30-34 with higher education (+)
Luxembourg	X_{17} – Computer skills (+)	X_2 – Internet activity (+)
Latvia	X_{10} – SME with innovations (-)	X_9 – Corporate R&D expenditures (-)
Malta	X_{14} – Teenagers quitting education (+)	X_9 – Corporate R&D expenditures (-)
Netherlands	X_{17} – Computer skills (+)	X_{13} – Young not working/ learning (-)
Poland	X_{10} – SME with innovations (-)	X_{14} – Teenagers quitting education (-)
Portugal	X_{10} – SME with innovations (-)	X_5 – HDI (-)
Romania	X_{14} – Teenagers quitting education (+)	X_{10} – SME with innovations (-)
Sweden	X_{11} – High internet skills (+)	X_{15} – Adults active in education (+)
Slovenia	X_{14} – Teenagers quitting education (-)	X_6 – PISA mathematics (+)
Slovakia	X_4 – PISA reading and interpret. (-)	X_{10} – SME with innovations (-)

While interpreting the list given in Table 2, one must keep in mind that positive values of some variables are in fact negative from the point of view of human social development. One example of such variable is X_{14} – *Teenagers who quit education*. This variable appeared six times among two most outlying variables for countries.

4. Results

Multidimensional Scaling was performed first on the full set of variables and then on its Flexible MDS proposed in the paper. Configuration of countries are presented on Figure 1. Blue circles mark the position of a country in all variables MDS, and red square show the position at Flexible MDS. Both points of the same country are joined with the line if they do not just overlap.

Figure 1. *Classical (blue circles) and Flexible (red squares) MDS results*
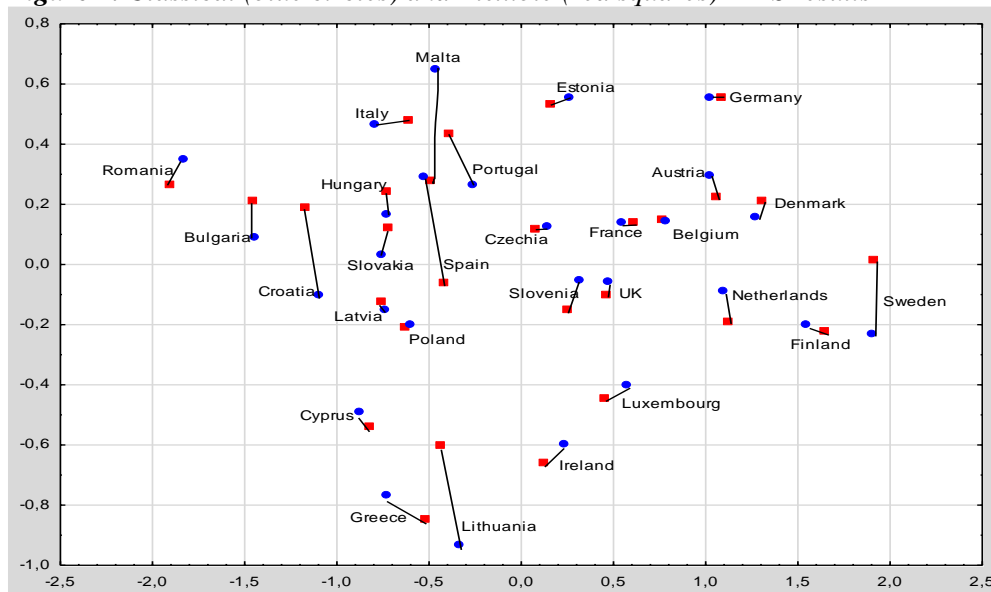
Source: Own composition.

The degree of flexibility measured by *M* equals to 0.7573. *M* compares distances in original MDS results (on the plane) with Flexible MDS. The M the more flexible is MDS, less vulnerable to changes in list of variables, and omitting "most extreme" one for each pair of objects. It is too early – with just one application – to propose qualitative interpretation to *M*.

5. Discussion

In Multidimensional Scaling, it is usually not easy to identify meanings/interpretation of both axes on the plane. In our example the meaning of horizontal axis seems rather obvious. Sokołowski and Markowska (2019) calculated Composite Smart Society Index, based on the same data that is used here. Table 3 presents the ranking of

countries together with the value of Composite Indicator, which takes value from [0;100] interval.

Table 3. *Composite Smart Society Index*

Position	Country	Composite Indicator	Position	Country	Composite Indicator
1	Sweden	85	15	Lithuania	44
2	Finland	78	16	Portugal	40
3	Netherlands	74	17	Spain	34
4	Denmark	73	17	Greece	34
5	Austria	68	19	Malta	33
6	Germany	67	19	Poland	33
7	Belgium	65	21	Italy	30
8	France	60	21	Latvia	30
9	Luxemburg	59	23	Cyprus	29
10	Great Britain	58	23	Slovakia	29
11	Slovenia	57	25	Hungary	28
12	Estonia	54	26	Croatia	24
12	Ireland	54	27	Bulgaria	14
14	Czech	50	28	Romania	3

Source: *Sokołowski and Markowska, 2019.*

It can be easily noticed that the distribution of countries along horizontal level on MDS graph closely follow the ranking presented in Table 3. Principal Component Analysis (PCA) with Varimax rotation has been performed to identify the vertical scale. With Kaiser's criterion on eigenvalues, four factors have been found. Loading is given in Table 4.

Table 4. *PCA loadings*

Variable	R&D	Internet	Skills	Education
X_1 – expenditures on R&D	***0.898***	0.003	0.371	0.024
X_2 – Internet activity	0.400	0.234	***0.827***	0.116
X_3 – Inventions in EPO	***0.831***	0.129	0.437	0.013
X_4 – PISA reading/interpr.	0.350	-0.277	0.581	0.452
X_5 – HDI	0.600	-0.038	***0.606***	0.328
X_6 – PISA maths	0.303	-0.508	***0.641***	0.135
X_7 – PISA science	0.360	-0.145	***0.797***	0.124
X_8 – R&D public spendings.	***0.828***	0.139	0.412	0.081
X_9 – R&D corporate expen.	***0.882***	-0.038	0,325	0.025
X_{10} – SME with innovations	***0.657***	-0.003	0.166	0.424
X_{11} – high internet skills	0.176	***0.856***	0.115	0.119
X_{12} – R&D employment	***0.747***	0.083	0.139	0.448
X_{13} – Young not work/learn	-0.202	-0.140	***-0.805***	-0.115
X_{14} – Teens quit. education	-0.131	-0.026	-0.047	***-0.851***
X_{15} – Adults active educat.	0.558	0.329	***0.650***	0.025
X_{16} – 30-34 higher educat.	-0.017	0.463	0.433	***0.668***
X_{17} – Computer skills	0.462	0.272	***0.741***	0.142

Variance explained	31.9%	9.4%	28.9%	11.6%

Source: Own calculations.

Loadings with module higher than 0.6 are reported in bold italics. Names for the four factors identified were imposed in Tab 3 to summarize the general meaning of factors. Coordinates of countries on MDS map were correlated with factor values. Correlation coefficients with respective p-values are given in Table 5. Variable X_{14} – *Teenagers qui. educations* are the most responsible for vertical coordinate on Figure 1 and on country position changes between classical and flexible MDS. Movements of countries are usually along vertical axis when changing from classical to flexible MDS.

Table 5. *Correlation coefficients and p-values (in brackets) between MDS coordinates and Factors*

Coordinate axis	R&D	Internet	Skills	Education
Classical horizontal	*0.695 (0.000)*	0.121 (0.541)	*0.664 (0.000)*	0.238 (0.222)
Classical vertical	0.133 (0.499)	*-0.491 (0.008)*	0.228 (0.243)	*-0.771 (0.000)*
Flexible horizontal	*0.731 (0.000)*	0..126 (0.524)	*0.627 (0.000)*	0.224 (0.252)
Flexible vertical	0.194 (0.324)	*-0.382 (0.045)*	0.127 (0.520)	*-0.726 (0.000)*

Source: Own calculations.

Variability along horizontal axis is much higher (range roughly from -2 to +2) than on vertical axis (from -1 to 0.8). It can relate to the portion of overall variance explained by the first and the third factor and their correlations.

6. Conclusions

The proposed procedure of Flexible Multidimensional Scaling (FMDS) is a new idea with the aim to introduce element of objectivity to the selection of variables for MDS. With this approach you can observe how vulnerable are you MDS results to slight changes in the list of variables. In FMDS distances between pair of objects are calculated based on the list of variables which can be not the same for each pair objects. The elimination of two variables is based on standardized modules, and distances are finally calculated as "per variable". The degree of flexibility can be assessed by the proposed measure.

Human Smart Social Development of EU countries serves not only as an example of FMDS application. With composite indicator and PCA the broader look into the problem has been offered. There is no definite and generally acceptable list of variables which characterize the analyzed phenomenon. In this sense, the flexibility approach helps to optimize the choice of variables. Out of the variables used in our study, the importance of percentage of teenagers quitting education is one of the most interesting findings in this research. For FMDS idea more applications are needed (or maybe some simulation studies) specially to scale flexibility measure *M*.

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